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II. Solution by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.; J. D. CRAIG, Frankfort, Ky.; and COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

Let A be the amount of P dollars for n years at r per cent. payable q times a year. Then $A=P[1+(r/q)]^{nq}=$

$$P\left[1+nr+\frac{n^2r^2}{1.2}-\frac{nr^2}{1.2.q}+\frac{n^3r^3}{1.2.3}+\frac{2n}{1.2.3q^2}-\frac{3n}{1.2.3.q}+\frac{n^4r^4}{1.2.3.4}+\ldots.\right].$$

Now if interest is to be compounded every instant, q is infinite and hence all terms in this series containing q will vanish, and we have

$$A=P\left[1+nr+\frac{n^2r^2}{1.2}+\frac{n^3r^3}{1.2.3}+\frac{n^4r^4}{1.2.3.4}+\ldots.\right]=Pe^{nr}.$$

$\therefore$ $A=\$1000(2.71828)^{.18}=\$1197.462+$.

III. Solution by D. G. DORRANCE, Jr., Camden, N. Y.

The formula for the amount of (a) dollars for (n) years at $r\%$ interest compounded every xth part of a year is

$$a\left(1+\frac{r}{x}\right)^{nx}$$

which expanded by the Binomial Theorem becomes

$$a\left(1+nx\frac{r}{x}+\frac{nx(nx-1)}{1.2}\frac{r^2}{x^2}+\frac{nx(nx-1)(nx-2)}{1.2.3}\frac{r^3}{x^3}+\text{etc.}\right)$$

which, when x is made infinitely large, becomes

$$a\left(1+nr+\frac{n^2r^2}{1.2}+\frac{n^3r^3}{1.2.3}+\frac{n^4r^4}{1.2.3.4}+\frac{n^5r^5}{1.2.3.4.5}+\text{etc.}\right).$$

Make $a=\$1000.$, $n=3$, and $r=.06$, and the above becomes

$\$1000(1+.18+.0162+.000972+.00004374+.00000157464+\text{etc.})$
$=\$1000(1.19721731464+)=\$1197.21731464+$, the required amount.

Also solved by *CHAS. C. CROSS*, and *ALOIS F. KOVARIK*.

ALGEBRA.

91. Proposed by NELSON S. RORAY, Professor of Mathematics, South Jersey Institute, Bridgeton, N. J.

Solve the following without making use of the determinant notation, and prove that the results obtained are the roots.

$$10x-2y+4z=5,$$
$$3x+5y-3z=7,$$
$$x+3y-2z=2.$$

I. Solution by H. C. WHITAKER, A. M., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.

Eliminate z : $12x+4y=9\ldots\ldots(1)$,

$$42x+14y=43\ldots\ldots(2).$$

Multiplying (1) by 7, and (2) by 2,

$$84x+28y=63,$$

$$84x+28y=86.$$

Whence the values of x, y, and z are infinite.

Multiply (1) by 43, and (2) by 9; equate results and reduce :

$$y=-3x.$$

Similarly, $z=-4x$.

The values $a/0$, $-(3a/0)$, $-(4a/0)$ prove in all three equations.

G. B. M. ZERR, *J. K. ELLWOOD*, *M. A. GRUBER*, and *ELMER SCHUYLER* each show that the equations are not simultaneous.

II. Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

Combining (1) and (3) we find $3x+y=\frac{9}{4}\ldots\ldots(4)$.

Combining (1) and (2) we find $3x+y=\frac{43}{14}\ldots\ldots(5)$.

Combining (2) and (3) we find $3x+y=8\ldots\ldots(6)$.

These three results are incompatible.

From (1)+(3) we get $15x-19y+18z=0$, or $15(x/z)-19(y/z)=-18$.

From (1)+(3)−(2) we get $8x-4y+5z=0$, or $8(x/z)-4(y/z)=-5$.

Solving these we find the ratios $x/z=-\frac{1}{4}$, $y/z=\frac{3}{4}\ldots\ldots(7)$.

The only answers to (4), (5), or (6) are $x=-\infty$, $y=+\infty$.

Similarly, $z=+\infty$, and the ratio of these infinities is given from (7) : $x:y:z::-1:3:4$.

Writing (1), $10(x/z)-2(y/z)+4=5/z=0$, since $z=\infty$.

$\therefore 10(-\frac{1}{4})-2(\frac{3}{4})=(-16/4)=-4$, which proves ; similarly for (2) and (3).

III. Solution by CHARLES C. CROSS, Libertytown, Md.

Take the general equations,

$$ax+by+cz=d,$$

$$a'x+b'y+c'z=d',$$

$$a''x+b''y+c''z=d''.$$

From which,

$$x=\frac{b'c''d+bc'd''+b''cd'-b'cd''-bc''d'-b''c'd}{ab'c''+a''bc'+a'b''c-a''b'c-a'bc''-ab''c'},$$

$$y=\frac{ac''d'+a'cd''+a''c'd-a''d'c-a'c''d-ad''c'}{D},$$

$$z=\frac{ab'd''+a'b''d+a''bd'-a''b'd-a'bd''-ab''d'}{D}.$$

Whence by substitution $x=\frac{23}{0}$, $y=-\frac{69}{0}$, and $z=-\frac{92}{0}$.

From this we see that a problem is impossible when

$$ab'c''+a''bc'+a'b''c-a''b'c-a'bc''-ab''c'=0.$$

IV. **Solution by J. M. BOORMAN, Woodmere, Long Island, N. Y.**

(1), (2), or (3) requires a *modulus*, otherwise it lacks one congruous root for some value of x, y, or z. If (1) and (3) be held congruous, modulus $(2)=-2.875$. $\therefore$ Constant $(2)=7+M=4.125$.

$$\left.\begin{array}{l}\therefore x=1.3,\ 1.4,\ \text{etc.};\ 2,\ 2.1\ldots\ldots\ldots\ldots\\ \ y=-1.65,\ -1.95,\ \text{etc.};\ -3.75,\ -4.05\ldots\ldots\\ \ z=-2.825,\ 3.225,\ \text{etc.};\ -5.625,\ -6.925\ldots\ldots\end{array}\right\}\begin{array}{l}\text{The law is obvious.}\\ \\ M=-2.875\ \ldots.(A).\end{array}$$

The above, with modulus $(1)=23$ taking (2) and (3) congruent, modulus $(3)=1.6\frac{3}{7}$. (1) and (2) taken congruent and the infinite series (B) and (C) are the required solution and roots.

Proof. $\frac{2}{3}(2)$ is $2x+3\frac{1}{3}y=4\frac{2}{3}+2z$; $\frac{1}{2}(1)$ is $5x-y=2.5-2z$. Subtracting (3) from the first of the above and adding it to the second, we have

$$x+\tfrac{1}{3}y=2.66\tfrac{2}{3} \text{ and } 6x+2y=4.5\ldots\ldots(4).$$
$$\underline{x+\tfrac{1}{3}y=0.75} \quad \text{or} \quad x+\tfrac{1}{3}y=0.75.$$

Defect of $2z$ in (2), $-1.91\frac{2}{3}$ of $3z=$modulus $(2)=-2.875$.

If $10x$, $-2y$, etc.$=$; ..... $\therefore y=-3.75$; $z=-5.625$ as above.
$(1)=20+7.5-22.5=5$.
$(2)=3x=6-18.75+16.875=7-M=4.125$.
$(3)\ x=2-11.25+11.25=2$.
Compute for $x=2.1$ for series (A) and its law.
Similarly, take (2) (3) congruent. $\therefore$ Modulus $(1)=23$. $\therefore$ (B).
Similarly, take (1) (2) congruent. $\therefore$ Modulus $(3)=1.6\frac{3}{7}$. $\therefore$ (C).

$$\begin{array}{ll}\therefore x_1=2,\ 2.2,\ 2.4,\ 2.6,\ 2.8,\ \text{etc.} & \text{Modulus }(1)=23.\\ \ y_1=2.\ 1.4.\ 0.8,\ 0.2,\ -0.4,\ \text{etc.} & \text{Constant }(1)=5+23=28.\\ \ z_1=3,\ 2.2,\ 1.4,\ 0.8,\ -0.2,\ \text{etc.} & \ldots\ldots(B).\\ \ x_2=2,\ 2.1.\ 2.2,\ \text{etc.} & \end{array}$$

$$\left.\begin{array}{ll} y_2=-2.9\frac{2}{7},\ -3.2\frac{2}{7},\ -3.5\frac{2}{7},\ \text{etc.} & \text{Modulus }(3)=1.6\frac{3}{7}.\\ z_2=-5.2\frac{1}{7},\ -5.6\frac{1}{7},\ 6.0\frac{1}{7},\ \text{etc.} & \text{Constant }(3)=36\frac{3}{7}.\end{array}\right\}\ \ldots\ldots\ldots(C).$$

Laws of (B) and (C) are obvious.

[Note on Problem 89. I have been interested with solutions of the problem "Solve $\left\{\begin{array}{l}x^2+y=7\\ x+y^2=11\end{array}\right\}$ by quadratics," the eleventh solution of which appeared in the February number of the Monthly.

This example was given me to solve about forty-three years ago, and hardly a year has passed since without having been requested to solve it. A so called quadratic solution appeared in Henkle's "Notes and Queries" about twenty-five years ago; and Professor Schuyler inserted substantially the same solution in the edition of his Algebra of 1884. Several solutions appeared in the mathematical columns of the *New England Journal of Education* about fifteen years ago.

All the solutions in which the values $x=2$ and $y=3$ are found by completing a square are *tentative* and depend indirectly upon the values sought. Take, for example, V. (*b*). $x^2+y=7$ is written $y-3=4-x^2$ and $x+y^2=11$ is written $y^2-9=2-x$. That is, first, y minus the value of y is equal to the square of the value of x minus the square of x; and second, y^2 minus the square of the value of y is equal to the value of x

minus x; and the ability to get an equation of which the square can be completed depends upon this operation,

In the seventh solution Dr. Zerr lets $t=3-y$ and gets $x^2-4=t$ and $x-2=t(3-y)$. Now the only integral value of t that will satisfy this assumption is zero; and this gives $x=2$ and $y=3$. And the the rest of the work is only necessary to show a little algebraic skill in getting a form the square of which can be completed. We are to observe, too, that when the square root is taken the double sign must be excluded.

In regard to the eleventh solution I ask how the solver got the $10ab=40+10b$ to subtract from (5)? I think he will admit that he found it by trial; or if not, he knew the values sought and from them found the member by which (4) must be multiplied to get this result. In every similar example this number is $2(y^2-x^2)(y>x)$. In the present example $2(9-4)=10$. Had the example been $\left\{\begin{matrix} x^2+y=\ 9 \\ x+y^2=27 \end{matrix}\right\}$ when $x=2$ and $y=5$, the number with which (4) must be multiplied is $2(25-4)=42$. So by knowing the roots in tnis mode of solution we always can find what to subtract to complete the square. But if we do not know the roots sought the finding of this quantity is a mere work *trial*; and would require much more labor than to find the values by inspection.

So I have come to the conclusion that when there are integral values for x and y they are best found by inspection, and the other values are best found by the use of Sturm's Theorem and Horner's Method of approximation. M. C. STEVENS.

Purdue University.

This problem, with four different solutions (all of which have been published in the MONTHLY) may be found in the *School Visitor*, Vol. III., pp. 114-115, and the editor, Mr. John S. Royer, calls it the "Yale Problem." Three solutions of this problem may also be found in *The Mathematical Visitor*, Vol. II., p. 3. and two solutions of it appeared in Vol. II., p. 25, of *The Analyst*.

$x^2+y=a$ is the equation of a parabola whose axis coincides with the Y-axis of reference, and its infinite branches extending in the negative direction; $x+y^2=b$ is the equation of a parabola whose axis coincides with the X-axis of reference and its infinite branches extending in the negative direction. These two curves may intersect in four points, intersect in two points and touch in one, intersect in two points, touch in one point or not intersect at all. The equations considered as simultaneous may, therefore, have four real roots, all different; four real roots, two equal and two different; two real roots, equal or different, and two imaginary roots; or four imaginary roots. The solution of the general case leads to a biquadratic which cannot be solved by quadratics. EDITOR F.]

GEOMETRY.

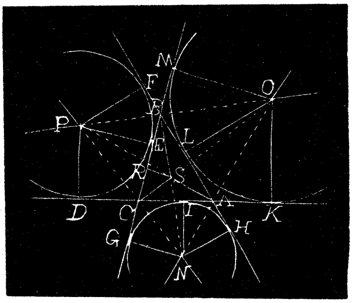
108. Proposed by NELSON L. RORAY, Bridgeton, N. J.

ABC is a triangle. O_1, O_2, O_3 centers of escribed circles. Prove altitudes of triangle $O_1O_2O_3$ are concurrent at center of incribed circle.

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Take the figure of my solution of Problem 97, Vol. V, No. 10, page 231, THE AMERICAN MATHEMATICAL MONTHLY.

We are to prove BN, AP, and CO the respective altitudes of $\triangle PON$, and passing through S, the center of inscribed circle of $\triangle ABC$.



$\triangle BGN$ and $\triangle BHN$ are equal; for $NG=NH$ (radii of same circle); $\angle BGN=\angle BHN$ =right triangle (radius to point of tangency), and BN is common.

$\therefore$ $\angle ABN=\angle CBN$; and as BN bisects $\angle ABC$, BN passes through S, the center of inscribed circle.

$\angle ABO+\angle ABN+\angle CBN+\angle CBP=$ two right triangles. But $\angle ABO=\angle CBP$ (property of escribed circles), and $\angle ABN=\angle CBN$. $\therefore$ $2\angle ABO=2\angle ABN=$two right angles. Whence $\angle ABO+\angle ABN=\angle NBO=$right triangle.